SAMPLED DATA OUTPUT FEEDBACK CONTROL OF TRI ROTOR UNMANNED AERIAL VEHICLE

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***ABSTRACT** — The research work in this paper deals with the control of the Tri rotor Unmanned Aerial Vehicle (UAV). The UAVs are attracting large interests from researchers and engineers due to versatility and usage, both in civil and military applications. Such vehicles are used for numerous applications by different organizations like law enforcement agencies, traffic police, coast guards and fire brigade units etc. The tri rotor VTOL UAV is one of the most efficient VTOL UAVs available and it can provide better flight time duration. The mathematical modelling of the tri rotor UAV is presented initially. To achieve stabilized flight for this UAV, sampled data sliding mode output feedback control design is used. Since the control systems are practically implemented digitally now-a-days, the control design is helpful as far as the practical implementation is concerned. Furthermore, practically we cannot afford sensors to measure all of the system states due to economic and technological constraints. Therefore there is a need to estimate the unknown states. The inertial coordinates and the yaw angle are the states that are assumed to be measured and for estimating the unknown states discrete-time observer is implemented. The overall control is achieved by applying the separation principle. The simulations show the performance of the discrete-time control of the UAV.*

Keywords - Tri Rotor Unmanned Aerial Vehicle, Sampled Data Output Feedback Control, Sliding Mode Control, Discrete-time Observer, Separation Principle, Discrete-time Control.

I INTRODUCTION

In the last few years, the UAVs have been a very active and vast research area. Miniature UAVs are widely used in various civil and military applications. Hovering and forward flight have forced the engineers & researchers to think beyond the classical helicopter. An alternate approach was proposed in the form of a quad rotor UAV. Its advantage over a conventional helicopter is that it doesn't require a swash plate. In order to gain more attributes at the expense of less cost, advancements have been made to come up with novel ideas.

Two multi-rotor UAVs were discussed in [1] and classical PID controllers were used to design the control systems. The first of these two UAVs is a tri rotor UAV. The UAV has three rotors so the reactive yaw torque is present. The effect of this torque is that it tends to rotate the body frame of the UAV. A solution to counter this yaw moment was to use a servo motor on one of the rotors. The servo motor tilts the rotor to nullify the yaw moment. This approach has also been discussed thoroughly in [2] and [24], where stabilization was achieved through a nested saturation control law. The other UAV presented in [1] is a coaxial tri rotor UAV and was proposed by Dragonfly Innovation Inc. (Saskatoon, Canada). It contains six rotors, so there is no need of using separate servo to counteract the yaw moment. However, the former configuration is more efficient from operational as well as economic point of view. It has higher maneuverability, ability to take sharp turns and better movement as compared to later configuration.

For control systems design, control is implemented on digital controllers (micro- controllers) now-a-days. Thus there is a need to develop the sampled data control of the UAV. It involves two approaches [3]. The first approach is to develop continuous control laws for the system and then discretize the continuous control by using a suitable sampling time. This approach is used by different researchers and control engineers [7], [8] and [9]. The sampling time is of much importance in digital control and should be as low as possible. But the sampling time can be reduced upto a certain extent only because the hardware available has its own limitations. More recently, DSP micro-controllers have enabled the achievement of control for more complex systems via greater sampling rates. The second approach is that we develop the equivalent discrete time model of the continuous system and then design the digital control laws depending on it. However, in case of nonlinear systems, exact discrete model is not available. Still the control can be designed depending upon approximate discrete-time model. This approach has an advantage over the first one is that it guarantees a larger region of attraction for the closed loop system with same sampling time [3]. It also possesses other advantages like high computational power, convenience and flexibility [17],18]. In this work, second approach will be considered for control design. To develop the approximate discrete model, the Euler differentiation approach is used. The control system of the tri rotor UAV is designed depending on approximate discrete-time model. One of the real-time implementation obstacles is the non-availability of all states in order to design a state feedback control law. Since one simply cannot afford sensors for all states due to economic and technological limitations [10]. The solution to this problem is the state estimator or observer. The observer is an online state estimator that estimates the unknown states and gives the information to the controller to stabilize the system. In [5], [11] and [12] sliding mode controller along with high gain observer was implemented and simulations were performed. The flight control of tri-propeller VTOL UAV was designed using the trajectory linearization control method in [19]. In [22], a linear optimal LQR controller was designed to control the attitude of tri rotor UAV. The different control techniques like PID controller, sliding mode controller and backstepping are used to control the attitude of multi-rotor UAVs [20], [23] and [24]. These techniques, mainly refer to continuous time control of multi-rotor UAVs which indicates the scarcity of discrete time control techniques [21]. The literature for the sampled data control of tri rotor UAV is not yet well developed.

In this work, a discrete-time sliding mode controller and discrete-time observer is proposed for the stabilization and estimation of the unknown states respectively. The measured states are the inertial coordinates and the yaw angle. According to the separation principle, the states which are used in state feedback control laws are replaced by their estimates coming from the discrete-time observer [4].

In this paper, the dynamic model of the tri rotor UAV is demonstrated via Euler-Lagrange Approach. Discrete sliding mode control laws are then designed on the basis of approximate discrete-time model to stabilize the UAV. Discrete-time observer is used to estimate the unknown states. This is performed by applying a diffeomorphism on the system dynamics and discretizing it. After applying the separation principle, the sampled data output feedback control system is simulated. Different reference signals will be given to the controllers and hence the overall performance is assessed with the observer estimating the states.

II DYNAMIC MODELING

The tri rotor has three rotors. Thrust is provided by the two main rotors and roll attitude is also stabilized by the difference in thrusts of these rotors. The third and tilting rotor controls the pitch attitude and also nullifies the yaw moment induced. Euler-Lagrange formalism is used to obtain the dynamic model of the tri rotor UAV [2], [20].

The states of this highly nonlinear system are given by the following vector: -

$$q = [\xi, \eta]^T = [x, y, z, \varphi, \theta, \psi]^T \qquad (1)$$

where [x, y, z] represent the inertial coordinates and [φ, θ, ψ] represent roll, pitch and yaw angles respectively.

The translational and rotational kinetic energies are represented by the following expressions:-

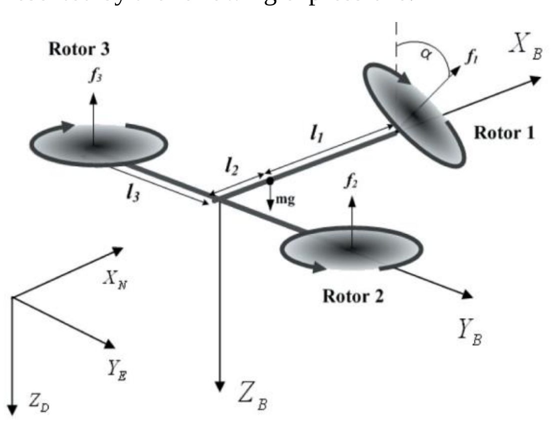

Fig.01 Reference Frames for a Tri Rotor VTOL UAV [1], [20]

$$T_{trans} = \frac{m}{2}\dot{\xi}^T\dot{\xi}$$

$$T_{rot} = \frac{1}{2}(\dot{\eta}^T J\dot{\eta})$$

The potential energy is described by the following relation:-

$$U = mgz$$

The Lagrangian is given by the following equation:-

$$L = T_{trans} + T_{rot} - U \qquad (2)$$

$$= \frac{m}{2}\dot{\xi}^T\dot{\xi} + \frac{1}{2}(\dot{\eta}^T J\dot{\eta}) - mgz$$

The above Lagrangian satisfies the following equation:-

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} = \boldsymbol{F}$$

$$\boldsymbol{F} = (\boldsymbol{F}_\xi, \boldsymbol{\tau}) \qquad (3)$$

where τ is the generalized moment and F_ξ represents the external forces acting on the body. By ignoring other forces of small magnitude, we now write:-

$$\hat{F} = \begin{pmatrix} 0 \\ 0 \\ u \end{pmatrix}$$

where,

$$u = f_1 + f_2 + f_3$$

$$f_i = k_i\omega_i^2$$

In above expression, f_i represents the thrust generated by i^{th} rotor. The variable ω_i is the angular speed and k_i is a constant.

Now transforming the $\hat{F}$ vector using the transformation matrix R, we get [2]:-

$$F_\xi = R\hat{F} \qquad (4)$$

where transformation matrix R is given by the following relationship:-

$$R = \begin{pmatrix} c\,\theta\, c\,\psi & -c\,\varphi\, s\,\psi + s\,\varphi\, s\,\theta\, c\,\psi & s\,\varphi\, s\,\psi + c\,\varphi\, c\,\psi\, s\,\theta \\ s\,\psi\, c\,\theta & c\,\varphi\, c\,\psi + s\,\psi\, s\,\theta\, s\,\varphi & s\,\psi\, s\,\theta\, c\,\varphi - s\,\varphi\, c\,\psi \\ -s\,\theta & c\,\theta\, s\,\varphi & c\,\theta\, c\,\varphi \end{pmatrix}^T$$

The terms $c\varphi$ and $s\varphi$ correspond to $\cos(\varphi)$ and $\sin(\varphi)$ respectively. Abbreviations are used here to avoid complexity. Now the generalized moments are described by the following expression:-

$$\boldsymbol{\tau} = \begin{pmatrix} \tau_\varphi \\ \tau_\theta \\ \tau_\psi \end{pmatrix} =$$

$$\begin{pmatrix} (f_2 - f_1)l_1 \\ f_3 l_2 \cos\alpha - m_3 g l_2 + (f_2 + f_1)l_1 \sin\gamma - 2m_2 g \sin\gamma \\ f_3 l_2 \sin\alpha \end{pmatrix} \qquad (5)$$

where l_i, α and γ are the system's parameters.

As there are no cross terms between the translational and rotational dynamics so, by separating these equations, we write:

$$m\ddot{\xi} + mg\boldsymbol{z} = \boldsymbol{F}_\xi \qquad (6)$$

$$J\ddot{\eta} + \dot{J}\dot{\eta} - \frac{1}{2}\frac{\partial y}{\partial \eta}(\dot{\eta}^T J\dot{\eta}) = \boldsymbol{\tau} \qquad (7)$$

Defining gyroscopic, centrifugal and Coriolis terms as:-

$$C(\eta,\dot{\eta})\dot{\eta} \triangleq \dot{J}\dot{\eta} - \frac{1}{2}\frac{\partial y}{\partial \eta}(\dot{\eta}^T J\dot{\eta})$$

The mathematical model of the tri rotor UAV can be written as:-

$$m\ddot{\xi} = u\begin{pmatrix} -s\theta \\ c\theta s\varphi \\ c\theta c\varphi \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -mg \end{pmatrix} \qquad (8)$$

$$J\ddot{\eta} = \boldsymbol{\tau} - C(\eta,\dot{\eta})\dot{\eta} \qquad (9)$$

where $\xi = [x \quad y \quad z]^T, \eta = [\varphi \quad \theta \quad \psi]^T$.

To facilitate the controller design, apply the following change of variables:-

$$\boldsymbol{\tau} = J\tilde{\boldsymbol{\tau}} + C(\eta,\dot{\eta})\dot{\eta}$$

Finally the rotational dynamics are written as:

$$\ddot{\varphi} = \tilde{\tau}_\varphi$$

$$\ddot{\theta} = \tilde{\tau}_\theta$$
$$\ddot{\psi} = \tilde{\tau}_\psi$$

III CONTROL DESIGN

The control laws for the tri rotor UAV are designed on the basis of approximate discrete-time model of the continuous nonlinear system. For that purpose, Euler differentiation is used. Let us consider a nonlinear system given by the following equation:-

$$\dot{x} = f(x,u) \quad (10)$$

Then by applying Euler differentiation, the following equation is achieved:

$$x[k+1] = x[k] + T(f(x[k],u[k])) \quad (11)$$

where T is the sampling time.

It should be noted that tri rotor UAV is an underactuated system with six degrees of freedom and four inputs. If we design the altitude and attitude controllers of the UAV only, it will inherently obtain position control by modulating appropriate attitude commands to closed loop for error [13]. So, attitude and altitude controllers will be designed to stabilize the system.

Attitude Control: To design the attitude control, roll angle controller is designed initially. The roll attitude dynamics of UAV are given by the following equation:-

$$\ddot{\varphi} = \tilde{\tau}_\varphi = u_2$$

Re-writing this equation in simple form, we get:-

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = u_2$$

By discretizing the above continuous differential equations, the following expressions are obtained:-

$$x_1(\mathrm{k}+1) = x_1(\mathrm{k}) + Tx_2(\mathrm{k})$$
$$x_2(\mathrm{k}+1) = x_2(\mathrm{k}) + Tu_2(\mathrm{k})$$

Defining the error by the following equation:-

$$e_1(\mathrm{k}) = x_{1d}(\mathrm{k}) - x_1(\mathrm{k})$$

From the continuous-time surface design approach discussed in [20], [20], we extend it to discrete time by considering the following Lyapunov function and performing the stability analysis:-

$$V(\mathrm{k}) = e_1^2(\mathrm{k})$$
$$\Delta V(\mathrm{k}) = V(\mathrm{k}+1) - V(\mathrm{k})$$
$$\Delta V(\mathrm{k}) = e_1^2(k+1) - e_1^2(\mathrm{k})$$
$$= (e_1(\mathrm{k}+1) + e_1(\mathrm{k}))\,(e_1(\mathrm{k}+1) - e_1(k))$$
$$= (e_1(k+1) + e_1(\mathrm{k}))\,(x_{1d}(\mathrm{k}+1) - Tx_2(\mathrm{k}) - x_{1d}(\mathrm{k}))$$

Assuming the following value of $Tx_2[k]$, we get the sliding manifold from equation (12):-

$$Tx_2(k) = \Delta x_{1d}(\mathrm{k}) + \alpha(e_1(\mathrm{k}+1) + e_1(k))$$
$$S_\varphi(k) = 0.61x_2(k) - \Delta x_{1d}(k) - \alpha(e_1(k+1) + e_1(k)) \quad (12)$$

where $\alpha > 0$ and $\Delta x_{1d}(\mathrm{k}) = x_{1d}(\mathrm{k}+1) - x_{1d}(\mathrm{k})$.

The trajectories should reach this surface in finite time and should be kept there for the time being. This is ensured by the following equations [20]:-

$$V(\mathrm{k}) = S_\varphi^2(\mathrm{k})$$
$$\Delta V(k) = S_\varphi^2(k+1) - S_\varphi^2(k) \quad (13)$$
$$= (S_\varphi(\mathrm{k}+1) + S_\varphi(\mathrm{k}))\,(S_\varphi(\mathrm{k}+1) - S_\varphi(k))$$
$$= (S_\varphi(\mathrm{k}+1) + S_\varphi(\mathrm{k}))\,\big(Tx_2(\mathrm{k}+1) - \Delta x_{1d}(\mathrm{k}+1) - \alpha(e_1(\mathrm{k}+2) - e_1(\mathrm{k})) - Tx_2(\mathrm{k}) + \Delta x_{1d}(\mathrm{k})\big)$$

After performing some algebraic manipulations, we obtain the following control law:-

$$u_2(\mathrm{k}) = -K_1 sat\left(S_\varphi(\mathrm{k}+1) + S_\varphi(\mathrm{k})\right) - K_2\left(S_\varphi(\mathrm{k}+1) + S_\varphi(\mathrm{k})\right) + Tc\alpha(e_1(\mathrm{k}+2) - e_1(k)) \quad (14)$$

where $\alpha > 0$, $K_1 = 0.1$, $K_2 = 10$, $T = 0.01$ and $c = 10$.

Similarly, the remaining attitude control laws are developed and are given below:-

$$u_3(k) = -K_1 sat\big(S_\theta(k+1) + S_\theta(k)\big) - K_2\big(S_\theta(k+1) + S_\theta(k)\big) + Tc\alpha(e_3(k+2) - e_3(k)) \quad (15)$$

$$u_4(k) = -K_1 sat\left(S_\psi(k+1) + S_\psi(k)\right) - K_2\left(S_\psi(k+1) + S_\psi(k)\right) + Tc\alpha(e_5(k+2) - e_5(k)) \quad (16)$$

Altitude Control: To design the altitude controller, consider the altitude dynamics of the UAV.

$$\ddot{z} = cos\theta cos\varphi \frac{u_1}{m} - g$$

Discretizing the altitude dynamics, we obtain:-

$$x_1(\mathrm{k}+1) = x_1(\mathrm{k}) + Tx_2(\mathrm{k})$$
$$x_2(\mathrm{k}+1) = x_2(\mathrm{k}) + T[\cos(\theta(k))\cos(\varphi(k))\frac{u_1(\mathrm{k})}{m} - g]$$

The following control law is proposed for the altitude tracking.

$$u_1(k) = \frac{m}{cos(\theta(k))\,cos(\varphi(k))}(g - K_1 sat\big(S_z(k+1) + S_z(k)\big) - K_2\big(S_Z(k+1) + S_z(k)\big) + Tc\alpha(e_7(k+2) - e_7(k))) \quad (17)$$

The control laws designed so far are capable to achieve a stable hover flight when necessary. The challenge now is that the states used in state feedback control laws are not available due to certain limitations. That is why there is a need to estimate the states by observer.

IV OBSERVER DESIGN

In this section, the discrete-time observer is designed on the basis of approximate discrete-time model. For that purpose, the continuous nonlinear model of the UAV is transformed into a canonical form which consists of a state affine linear part and a nonlinear part by applying a diffeomorphism [5], [14] and [15]. After applying the change of coordinates, the approximate discrete-time model is obtained and discrete-time observer is designed on its basis. Estimated states are used to provide the required control action.

Inertial coordinates (x,y) are assumed to be known along with altitude (z) and yaw angle (ψ). Differential global positioning system (DGPS) provides accurate information regarding position and velocity [5], [10]. Altitude is measured with the help of an altimeter [16]. Digital compass will help to get information about the yaw attitude.

The mathematical model of the tri rotor UAV can be written as:-

$$\ddot{x} = -\frac{u_1}{m} sin\theta$$

$$\ddot{y} = \frac{u_1}{m} cos\theta sin\varphi$$
$$\ddot{z} = cos\theta cos\varphi \frac{u_1}{m} - g$$
$$\ddot{\varphi} = \tilde{\tau}_\varphi$$
$$\ddot{\theta} = \tilde{\tau}_\theta$$
$$\ddot{\psi} = \tilde{\tau}_\psi$$

Applying the transformation $Q = \phi(X)$, the above system is transformed into canonical form with a part of it depending on the thrust input u_1. The change of coordinates is as follows [5]:-

$$Q = [Q_1 \;\; Q_2 \;\; Q_3 \;\; Q_4]^T$$
$$Q_j = [Q_{j1} \;\; Q_{j2} \;\; Q_{j3} \;\; Q_{j4}]^T, \qquad j = 1,2$$
$$Q_i = [Q_{i1} \;\; Q_{i2}]^T, \qquad i = 3,4$$

The new system's states are given by: -

$Q_{11} = x$	$Q_{21} = y$
$Q_{12} = \dot{x}$	$Q_{22} = \dot{y}$
$Q_{13} = -sin\theta$	$Q_{23} = cos\theta sin\phi$
$Q_{14} = -\dot{\theta} cos\theta$	$Q_{24} = \dot{\varphi} cos\theta cos\phi - \dot{\theta} sin\theta sin\varphi$
$Q_{31} = z$	$Q_{41} = \psi$
$Q_{32} = \dot{z}$	$Q_{42} = \dot{\psi}$

The first measured output is x. The non-measurable signals are obtained by differentiating it.

$$y_1 = x = Q_{11}$$
$$\dot{Q}_{11} = Q_{12} = \dot{x}$$
$$\dot{Q}_{12} = \ddot{x} = -\frac{u_1}{m} sin\theta = Q_{13} \frac{u_1}{m}$$
$$\dot{Q}_{13} = -\dot{\theta} cos\theta = Q_{14}$$
$$\dot{Q}_{14} = \dot{\theta}^2 sin\theta - \tilde{\tau}_\theta cos\theta = \rho_{14}(Q,u)$$

The first transformed system related to the first output is collectively written as:-

$$\dot{Q}_1 = A_1(u)Q_1 + \rho_1(Q,u) \qquad (18)$$
$$Y_1 = C_1 Q_1$$

where,

$$A_1(u) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{u_1}{m} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad \rho_1(Q,u) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \rho_{14}(Q,u) \end{bmatrix}$$
$$C_1 = [1 \quad 0 \quad 0 \quad 0]$$

The second measured output is y. The non-measurable signals are obtained by differentiating it successively. The respective canonical form will be obtained as follows:-

$$y_2 = y = Q_{21}$$
$$\dot{Q}_{21} = Q_{22} = \dot{y}$$
$$\dot{Q}_{22} = \ddot{y} = \frac{u_1}{m} cos\theta sin\varphi = Q_{23} \frac{u_1}{m}$$
$$\dot{Q}_{23} = \dot{\varphi} cos\theta cos\varphi - \dot{\theta} sin\theta sin\varphi = Q_{24}$$
$$\dot{Q}_{24} = \tilde{\tau}_\varphi cos\theta cos\varphi - 2\dot{\varphi}\dot{\theta} sin\theta cos\varphi - \dot{\varphi}^2 cos\theta sin\varphi - \tilde{\tau}_\theta sin\theta sin\varphi - \dot{\theta}^2 cos\theta sin\varphi = \rho_{24}(Q,u)$$

The collective form is written as below:-

$$\dot{Q}_2 = A_2(u)Q_2 + \rho_2(Q,u) \qquad (19)$$
$$Y_2 = C_2 Q_2$$

where,

$$A_2(u) = A_1(u) \qquad \rho_2(Q,u) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \rho_{24}(Q,u) \end{bmatrix}$$
$$C_2 = C_1$$

For the third and fourth outputs z and ψ, the collective form and the canonical matrices are given as follows:-

$$\dot{Q}_i = A_i(u)Q_i + \rho_i(Q,u) \qquad (20)$$
$$Y_i = C_i Q_i$$

where $i = 3, 4$

$$A_i(u) = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \qquad C_i = [1 \quad 0]$$
$$\rho_i(Q,u) = \begin{bmatrix} 0 \\ \rho_{i2}(Q,u) \end{bmatrix}$$
$$\rho_{32}(Q,u) = cos\theta cos\varphi \frac{u_1}{m} - g \qquad \rho_{42}(Q,u) = \tilde{\tau}_\psi$$

Combining $\dot{Q}_1, \dot{Q}_2, \dot{Q}_3$ and $\dot{Q}_4$ into a single collective equation to represent the transformed dynamical model of the tri rotor UAV, we obtain:-

$$\dot{Q} = A(u)Q + B\rho(Q,u) \qquad (21)$$
$$Y = CQ$$

where,

$$A(u) = diag\big(A_1(u), A_2(u), A_3(u), A_4(u)\big)$$
$$B = diag([0 \quad 0 \quad 0 \quad 1]^T, [0 \quad 0 \quad 0 \quad 1]^T, [0 \quad 1]^T, [0 \quad 1]^T)$$
$$\rho(Q,u) = [\rho_{14}(Q,u), \rho_{24}(Q,u), \rho_{32}(Q,u), \rho_{42}(Q,u)]^T$$
$$C = diag(C_1, C_2, C_3, C_4)$$

The following continuous time observer to estimate the unknown states is proposed:-

$$\dot{\hat{Q}} = A(u)\hat{Q} + B\rho\big(\hat{Q}, u\big) + H(Y - C\hat{Q}) \qquad (22)$$

The observer gain H is chosen as:-

$$H = diag(H_1, H_2, H_3, H_4)$$

However, since the observer is implemented digitally these days, there is a need to discretize the above proposed observer. Using Euler differentiation to get the approximate discrete-time model of above equation we get:

$$\begin{aligned}\hat{Q}(\mathrm{k}+1) &= \hat{Q}(\mathrm{k}) + TA\big(u(\mathrm{k})\big)\hat{Q}(\mathrm{k}) + TB\rho\left(\hat{Q}(\mathrm{k}), u(\mathrm{k})\right) + H_T(Y(\mathrm{k}) - C\hat{Q}(\mathrm{k})) \\ &= \big[I + TA\big(u(\mathrm{k})\big)\big]\hat{Q}(\mathrm{k}) + TB\rho\left(\hat{Q}(\mathrm{k}), u(\mathrm{k})\right) + H_T(Y(\mathrm{k}) - C\hat{Q}(\mathrm{k})) \\ &= A_{T,d}\big(u(k)\big)\hat{Q}(k) + B_{T,d}\rho\left(\hat{Q}(k), u(k)\right) + H_T(Y(k) - C\hat{Q}(k)) \qquad (23)\end{aligned}$$

Now the discretized gains are chosen in such a way that the eigenvalues of $A_{T,d}(u(\mathrm{k})) - B_{T,d}H_T$ lie inside the unit circle. The values of observer gains are:-

$$H_1 = [2,8,8,8]^T \qquad H_2 = [2,10,10,8]^T$$
$$H_3 = [2,10]^T \qquad H_4 = [2,10]^T$$

These values ensure the position of eigenvalues inside the unit circle. The above observer will estimate the unknown states of the transformed system. To get the state information of the original system, following transformation is applied:-

$$X = \phi^{-1}(Q)$$

$x = Q_{11}$	$y = Q_{21}$
$\dot{x} = Q_{12}$	$\dot{y} = Q_{22}$
$\theta = -sin^{-1}(Q_{13})$	$\varphi = sin^{-1}(\frac{Q_{23}}{cos\theta})$
$\dot{\theta} = -\frac{Q_{14}}{cos\theta}$	$\dot{\varphi} = \frac{Q_{24} + \dot{\theta}sin\theta sin\varphi}{cos\theta cos\varphi}$
$z = Q_{31}$	$\psi = Q_{41}$
$\dot{z} = Q_{32}$	$\dot{\psi} = Q_{42}$

The simulation results show the effectiveness of above designed discrete sliding mode controller and discrete-time observer. Once the states are estimated, they are fed to the state feedback controller to design the required inputs for the system to stabilize or track the reference signal as required.

V SIMULATION RESULTS

To check the above proposed controller and observer design, simulations are performed for the closed loop system. The measured states are fed to the observer to estimate the unknown states and the estimated states are given to the controller to stabilize the system according to separation principle. The values of controller and observer gains are specified in their respective sections. The system parameters, mass and gravitational acceleration, have the value of 1 kg and 9.8 ms-2. Initial conditions for the tri rotor UAV system and the observer are all assumed to be zero. For the attitude controllers, time-varying (sine wave) reference signals are provided to track. The following tracking command is given to altitude controller:-

$$z_d = 20\ m$$

The error in tracking the desired roll angle is depicted in figure 2. In figure 3 the desired yaw angle, which was a sinusoid with a bias, tracking is shown along with the error. The error observed while tracking a reference pitch angle is shown in figure 5. The control inputs U2 and U3 are shown in figure 4. The altitude tracking with its respective control input is depicted in figure 6. Figure 7 shows the true attribute of SMC, which is the robustness to parameter uncertainties and disturbances. The wind gusts were used to assess the controller's performance.

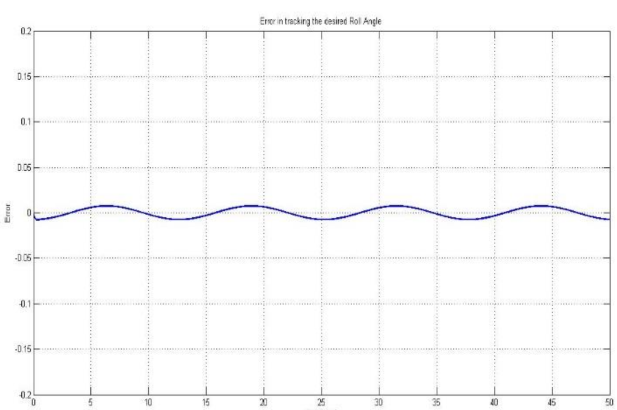

Fig.02 Error in Tracking Desired Roll Angle

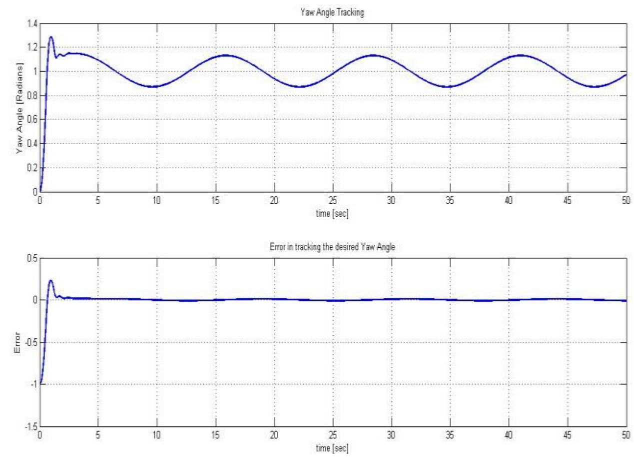

Fig.03 Yaw Angle Tracking and Error

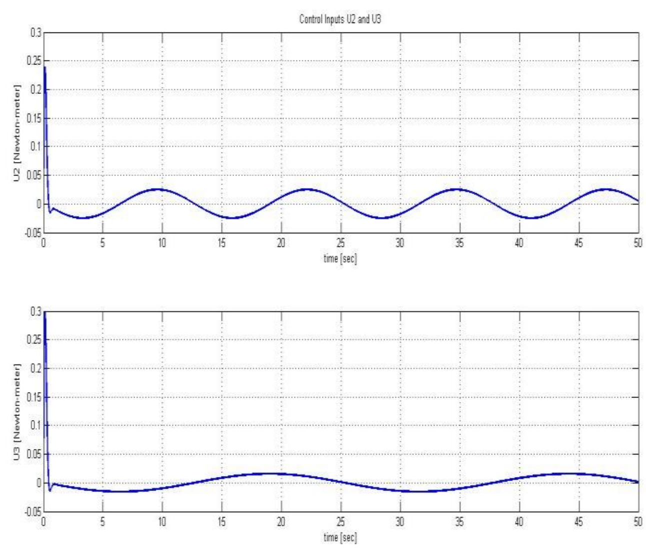

Fig.04 Control Inputs U2 and U3

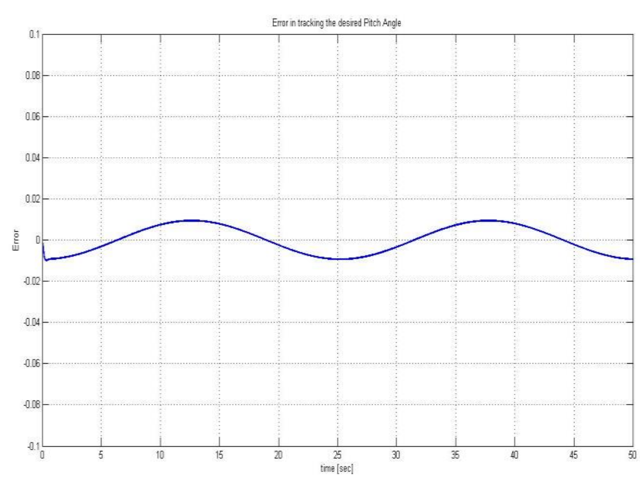

Fig.05 Tracking Error in Pitch Angle

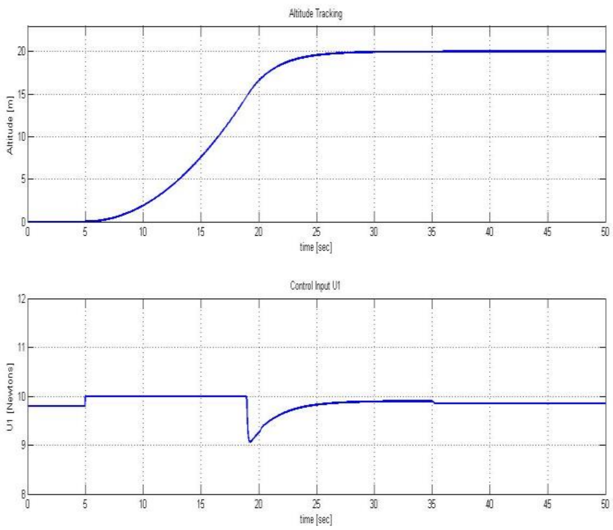

Fig.06 Altitude Tracking and Control Input U1

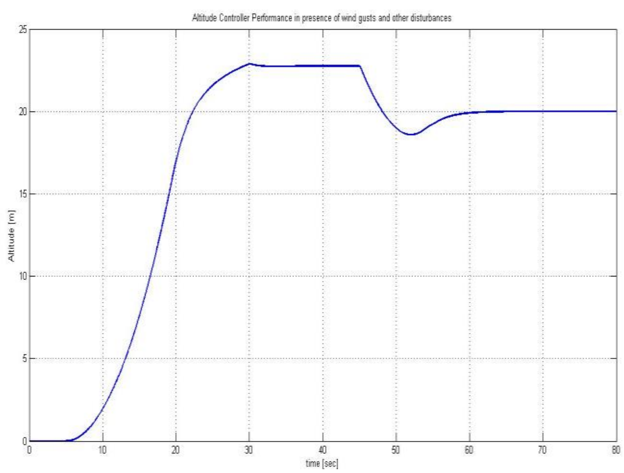

Fig.07 Robustness to Disturbances and Wind Gusts

VI CONCLUSIONS

In this paper, discrete-time control laws have been presented for stabilization and tracking purposes of the tri rotor VTOL UAV. The state observer is also presented to estimate the unknown states. Initially, discrete control laws based on sliding mode control technique were proposed by using approximate discrete-time model of the system. Then, the original continuous time model of UAV was transformed into another continuous model of the same system via diffeomorphism. The transformed mathematical model was used to design a continuous time observer. The continuous observer was discretized using Euler differentiation to obtain the discrete-time observer.

The proposed controllers have shown good performance as compared to other controllers which were discussed in [1] because they stabilized the given system under specific initial conditions. In [2], the nested control laws were simply discretized to control the UAV, which according to [3], [17] and [18] is not a well-developed approach because it has a limited region of attraction as compared to discretized model based approach. The controller proposed in this research also takes care of the parameter uncertainties and disturbances which was not discussed in [1] and [2].

Additionally the discrete sliding mode controller has shown an overall good performance keeping the errors within certain bounds. In case of the step reference signal and sinusoid with a bias, this error is reduced asymptotically. Furthermore the altitude controller also performed well in the presence of parameters uncertainties and disturbances like wind gusts. The discrete-time observer has also been able to handle the measurement noise.

As far as the experimental results are concerned, these discrete controllers and observer are easy to implement practically on microcontrollers, DSP kits and FPGAs with good results as described by simulations.

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